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False Vacuum Decay after Inflation

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Abstract

Inflation is terminated by a non-equilibrium process which finally leads to a thermal state. We study the onset of this transition in a class of hybrid inflation models. The exponential growth of tachyonic modes leads to decoherence and spinodal decomposition. We compute the decoherence time, the spinodal time, the size of the formed domains and the homogeneous classical fields within a single domain.

An inflationary phase in the early history of the universe can explain its present flatness and homogeneity as well as the anisotropy of the cosmic microwave background [1]. Inflation has to terminate in some non-equilibrium process which eventually leads to a thermal state. This is elegantly realized in hybrid models of inflation where the time evolution of the inflaton field φ controls the transition from a false vacuum state to the true vacuum [2]. Hybrid inflation arises naturally in supersymmetric theories with spontaneous symmetry breaking [3]. The dynamics of the inflaton field during the inflationary phase is then determined either by supergravity corrections [3], by quantum corrections [4] or by the effects of the supersymmetry breaking sector of the theory [5].

The phase transition at the end of inflation is of crucial importance since it determines the initial conditions for the further evolution towards the thermal state. Important quantities, which have to be obtained, are the reheating temperature and the initial abundances of weakly coupled particles, such as gravitinos, which are not in thermal equilibrium. In hybrid inflation the decay of the false vacuum takes place via spinodal decomposition [6], similar to a second-order phase transition. Such a decay has already been discussed in the literature in the context of new inflation [7, 8] and also for scalar ϕ^4 -theory in Minkowsky space [9].

In the following we shall study the onset of this transition in a class of hybrid inflation models [5], where the evolution is controlled by the slow-roll motion of the inflaton field. This will allow us to follow in detail the breakup of the homogeneous inflationary phase into domains and to compute the decoherence time, the spinodal time, the average size of the formed domains and the homogeneous classical fields inside a single domain.

False vacuum inflation

Hybrid inflation can be based on the superpotential [5]

$$W = W_G + W_S = \lambda T \left(M_G^2 - \Sigma^2 \right) + M_S^2 (\beta + S) . \quad (1)$$

Here W_G describes the supersymmetric breaking of a global symmetry, whereas the Polonyi superpotential W_S accounts for supersymmetry breaking [10]. The discrete Z_2 symmetry of the model leads to the well known domain wall problem [11]. We consider this case only for simplicity; the extension to a continuous $U(1)$ symmetry, where no such problems exist, is straightforward. The mass M_S is the scale of supersymmetry breaking, associated with the gravitino mass $m_{3/2} = M_S^2/M_P \exp(2 - \sqrt{3})$, and the constant $\beta \simeq (2 - \sqrt{3})M_P$, where $M_P = (8\pi G_N)^{-1/2} = 2.4 \times 10^{18} \text{ GeV}$ is the Planck mass. Successful inflation requires $M_S^2 \ll \lambda M_G^2$. A rotation of the fields T and S by the small angle

$\xi \simeq M_S^2/(\lambda M_G^2)$ changes the superpotential (1) to the standard form of an O’Raifeartaigh model [12].

In the case of global supersymmetry one obtains from the superpotential (1) the scalar potential

$$V_0 = \lambda^2 |M_G^2 - \Sigma^2|^2 + 4\lambda^2 |T|^2 |\Sigma|^2 + M_S^4 , \quad (2)$$

with the ground state

$$\langle T \rangle = 0 , \quad |\langle \Sigma \rangle| = M_G . \quad (3)$$

For $|T|^2 > M_G/2$, V_0 has a local minimum at $\Sigma = 0$ where it is flat in S and T .

Supergravity corrections are included in the potential

$$V = e^{K/M_P^2} \left[\left| \frac{\partial W}{\partial z_i} + \frac{z_i^* W}{M_P^2} \right|^2 - 3 \frac{|W|^2}{M_P^2} \right] , \quad (4)$$

where $z_i = T, \Sigma, S$, $i = 1 \dots 3$. We choose K to be the canonical Kähler potential, $K = \sum_i |z_i|^2$, which is singled out in $N = 2$ supersymmetry [13]. Up to corrections $\mathcal{O}(1/M_P^n)$, the expectation values of the three fields T , Σ and S in the true vacuum are

$$\langle T \rangle = 0 , \quad |\langle \Sigma \rangle| = M_G , \quad \langle S \rangle = (2 - \sqrt{3})M_P . \quad (5)$$

The supergravity corrections distinguish between the real and the imaginary part of $T \simeq (\varphi + i\chi)/\sqrt{2}$. For $\varphi > \varphi_c = M_G$, V has again a local minimum at $\Sigma = (\sigma + i\rho)/\sqrt{2} = 0$. The energy density is then $V \simeq \lambda^2 M_G^4$, which leads to an inflationary phase with Hubble parameter $H = \lambda M_G^2/(\sqrt{3}M_P)$. Here the potential is almost flat in φ which plays the role of the inflaton. The mass of ρ is large, $m_\rho \simeq 2\lambda\varphi > 2\lambda M_G$. The minimum with respect to S and χ lies at $S = \chi = 0$, but the corresponding masses are small, $m_S = \mathcal{O}(H)$ and $m_\chi = \mathcal{O}(H\varphi/M_P)$.

In this paper we are interested in the decay of the false vacuum driving inflation, which takes place near the critical point $\varphi_c = M_G$. This phase transition is determined by the dynamics of the lightest scalar fields, the inflaton φ and σ , which becomes massless at φ_c . The scalar potential for φ and σ reads, up to corrections $\mathcal{O}(1/M_P^2)$,

$$V(\varphi, \sigma) = \lambda^2 \left(M_G^2 - \frac{\sigma^2}{2} \right)^2 + \lambda^2 \varphi^2 \sigma^2 + \kappa \varphi . \quad (6)$$

The linear term in the potential is $\mathcal{O}(1/M_P)$, with

$$\kappa = 2^{3/2}(2 - \sqrt{3}) \lambda \frac{M_S^2 M_G^2}{M_P} \simeq 0.6 \lambda m_{3/2} M_G^2 . \quad (7)$$

For a certain range of parameters the time evolution of the inflaton field during inflation is determined by the linear term in the potential [5]. Representative values are $M_S \simeq 1.6 \times 10^{10}$ GeV, which implies $m_{3/2} \simeq 130$ GeV, $\lambda \simeq 1.3 \times 10^{-5}$ and $M_G \simeq 10^{16}$ GeV. This yields $H \simeq 3.1 \times 10^8$ GeV and $\kappa \simeq (4.6 \times 10^9 \text{ GeV})^3$. From the full supergravity potential (4) one obtains for the slow-roll conditions of the inflaton φ , as long as σ^2 is sufficiently small,

$$\epsilon = \frac{M_{\text{P}}^2}{2} \left(\frac{V'}{V} \right)^2 = \frac{4\xi^2 \beta^2}{M_{\text{P}}^2} \left(1 - \frac{\varphi^3}{2\sqrt{2}\xi\beta M_{\text{P}}^2} + \dots \right) \ll 1, \quad (8)$$

$$\eta = M_{\text{P}}^2 \frac{V''}{V} = \frac{3}{2} \frac{\varphi^2}{M_{\text{P}}^2} + \dots \ll 1. \quad (9)$$

These conditions are clearly satisfied for values of $\varphi \ll M_{\text{P}}$ and in particular below the critical value φ_c . The solution of the equation of motion for the homogeneous classical field reads,

$$\varphi_{cl}(t) = M_G - \frac{\kappa}{3H} t, \quad (10)$$

where we have fixed the origin of time by $\varphi(0) = \varphi_c = M_G$. The transition from the inflationary phase to the thermal phase starts at $t = 0$.

Fluctuations near the critical point

At $\varphi = \varphi_c$ the mass term of the σ -field changes sign. This is analogous to a second-order phase transition with $\varphi(t)$ playing the role of a time dependent temperature of the thermal bath. In flat de Sitter space with metric

$$ds^2 = dt^2 - e^{2Ht} d\vec{x}^2, \quad (11)$$

the quantum fields $\varphi(t, \vec{x})$ and $\sigma(t, \vec{x})$ can be expanded into plane waves which are momentum eigenstates,

$$\varphi(t, \vec{x}) = \varphi_{cl}(t) + e^{-\frac{3}{2}Ht} \int \frac{d^3k}{(2\pi)^{3/2}} \left(a_{\varphi}(\vec{k}) \varphi_k(t) e^{i\vec{k}\vec{x}} + a_{\varphi}^{\dagger}(\vec{k}) \varphi_k^*(t) e^{-i\vec{k}\vec{x}} \right), \quad (12)$$

$$\sigma(t, \vec{x}) = e^{-\frac{3}{2}Ht} \int \frac{d^3k}{(2\pi)^{3/2}} \left(a_{\sigma}(\vec{k}) \sigma_k(t) e^{i\vec{k}\vec{x}} + a_{\sigma}^{\dagger}(\vec{k}) \sigma_k^*(t) e^{-i\vec{k}\vec{x}} \right). \quad (13)$$

Note the absence of $\sigma_{cl}(t)$, the homogeneous part of $\sigma(t, \vec{x})$. This is due to the initial condition $\sigma_{cl}(0) = 0$ at the end of inflation. The annihilation and creation operators satisfy canonical commutation relations,

$$[a_{\varphi}(\vec{k}), a_{\varphi}^{\dagger}(\vec{k}')] = [a_{\sigma}(\vec{k}), a_{\sigma}^{\dagger}(\vec{k}')] = \delta^3(\vec{k} - \vec{k}'), \quad (14)$$

and the canonically conjugate operators to the fields φ and σ are

$$\pi_\varphi(t, \vec{x}) = e^{3Ht} \dot{\varphi}(t, \vec{x}) , \quad \pi_\sigma(t, \vec{x}) = e^{3Ht} \dot{\sigma}(t, \vec{x}) . \quad (15)$$

The assumption of a constant Hubble parameter is well justified during inflation and the onset of the subsequent phase transition.

Near the critical point $\varphi = \varphi_c$ the potential for the quantum fields φ and σ takes the form,

$$V(\varphi, \sigma) = -\frac{1}{2} D^3 t \sigma^2 + \dots . \quad (16)$$

where $D^3 = 4\lambda^2 \kappa M_G / (3H)$. For the parameters given above one has $D = 3.4H$. Note, that this equation also holds in other inflationary models where the inflaton is slowly rolling near $\varphi = \varphi_c$, since $D^3 t$ is the first term in an expansion of the σ -mass around $t = 0$.

The mode functions $\varphi_k(t)$ and $\sigma_k(t)$ satisfy the equations of motion,

$$\ddot{\varphi}_k + \left(k^2 e^{-2Ht} - \frac{9}{4} H^2 \right) \varphi_k = 0 , \quad (17)$$

$$\ddot{\sigma}_k + \left(k^2 e^{-2Ht} - \frac{9}{4} H^2 - D^3 t \right) \sigma_k = 0 . \quad (18)$$

The solution of eq. (17) is well known from studies of density fluctuations during inflation. It is given by a Hankel function

$$\varphi_k(t) = \frac{1}{2} \left(\frac{\pi}{H} \right)^{1/2} H_{3/2}^{(1)}(z) , \quad (19)$$

where $z = k \exp(-Ht)/H$ and

$$H_{3/2}^{(1)}(z) = - \left(\frac{2}{\pi z} \right)^{1/2} e^{iz} \frac{z + i}{z} . \quad (20)$$

The solution of eq. (18) has to be determined numerically. However, asymptotically, for $t \ll t_k$ and $t \gg t_k$, where

$$k^2 e^{-2Ht_k} - \frac{9}{4} H^2 = |D^3 t_k| , \quad (21)$$

useful approximate analytical solutions can be found¹. At large negative times we again have the Hankel function analogous to (19) as solution. This is the de Sitter-space initial condition.

¹For momenta $3H/2 < k < k_{max} \simeq (D^3/(2He))^{1/2} \simeq 3H$ eq. (21) has more than one solution for t_k . We choose the smallest value for t_k to construct an approximate solution. For a more detailed discussion, see [14].

For $t \gg t_k$ eq. (18) reduces to

$$\ddot{\sigma}_k - D^3 t \sigma_k = 0 . \quad (22)$$

which describes a scalar field in Minkowski space with mass squared $\mu^2(t) = -D^3 t$, decreasing with time. The solutions are given by the Airy functions Ai and Bi [15]. An approximate analytic solution of eq. (18) can be obtained by matching Airy and Hankel functions at $t = t_k$,

$$\sigma_k(t) = C_A(k) \text{Ai}(z) + C_B(k) \text{Bi}(z) , \quad (23)$$

where $z = Dt$. For soft momenta, $k < H$, one finds

$$C_A = i\sqrt{\frac{\pi}{2D}} , \quad C_B = \sqrt{\frac{\pi}{2D}} . \quad (24)$$

For $z = Dt \gg 1$ the behaviour of $\varphi_k(t)$ is dominated by the asymptotic growth of Bi(z),

$$\text{Bi}(z) \simeq \frac{1}{\sqrt{\pi}} z^{-1/4} \exp\left(\frac{2}{3} z^{3/2}\right) . \quad (25)$$

The physical meaning of such tachyonic instabilities was studied in detail by Guth and Pi for fixed negative mass squared, $\mu^2(t) = -const$, in the context of new inflation [7]. More recently, the dynamics of spinodal decomposition has been investigated by several groups [8, 9].

The situation considered here is similar to the previously discussed cases with respect to the occurrence of spinodal decomposition. However, a crucial difference is that the negative curvature of the potential is neither constant nor suddenly ‘turned on’. The change from positive to negative curvature is, on the contrary, controlled by the slow-roll of the inflaton field across the critical point $\varphi = \varphi_c$.

An important quantity is the growth of the total fluctuation, the variance

$$\langle \sigma^2(t) \rangle_r = \langle \sigma^2(t) \rangle - \langle \sigma^2(0) \rangle , \quad (26)$$

where

$$\langle \sigma^2(t) \rangle = \int \frac{d^3 k}{(2\pi)^3} e^{-3Ht} |\sigma_k(t)|^2 \equiv \int dk f_k(t) . \quad (27)$$

Results for the spectral function $f_k(t)$ are shown in Fig. 1. The expected rapid growth with time is clearly visible. For small k one has $f_k \propto k^2$, since $|C_B(k)| \simeq const$. For larger momenta, $k > k_* \sim 3H$, also the time t_k is large, which leads to a delayed growth of these modes and a rapid fall-off of $f_k(t)$ at large k . For very large momenta, $k \gg k_*$, one sees the behaviour of a massless field, i.e. $f_k \propto k$, which is redshifted with increasing time due to the Hubble expansion.

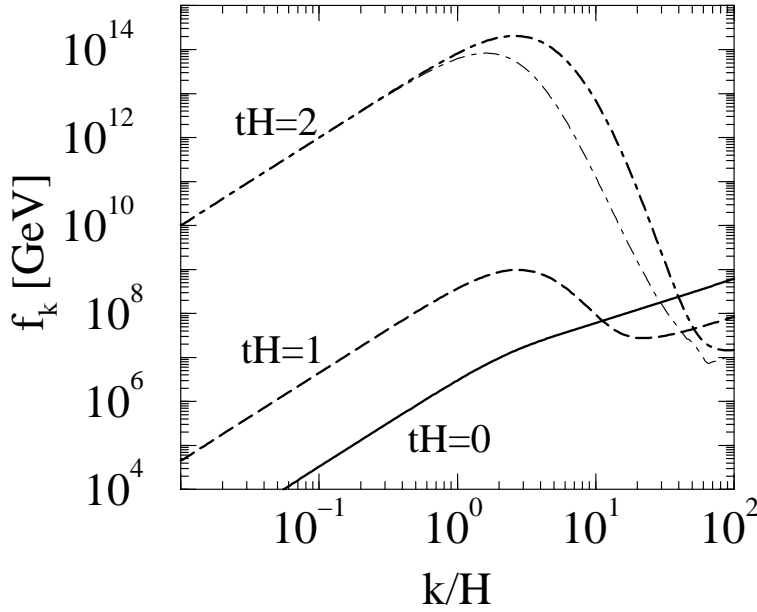


Figure 1: *Spectral function $f_k(t)$ of the variance $\langle \sigma^2(t) \rangle_r^{1/2}$. Thick lines correspond to numerical solutions; the thin line denotes an approximate analytical solution.*

The growth of the variance $\langle \sigma^2(t) \rangle_r$ is shown in Fig. 2. It clearly exhibits the characteristic feature of our model, a faster than exponential growth with time. Eventually, this growth is terminated by the back reaction of the produced fields. The spinodal time at which the formation of domains is completed can be estimated by comparing the variance with the global minimum of the potential,

$$\langle \sigma^2(t_{sp}) \rangle_r \simeq 2M_G^2. \quad (28)$$

From Fig. 2 one reads off $t_{sp} \simeq 3/H$.

Tachyonic modes σ_k become classical at the decoherence time t_{dec} . At later times the system is characterized by a classical probability distribution $f(\sigma_k, t)$ for the amplitude σ_k . The homogeneous mode of the system $\sigma_{cl}(t) = \langle \sigma(t, \vec{x}) \rangle$ remains zero after spinodal decomposition [16].

The decoherence time t_{dec} is determined by the requirement that the tachyonic modes become classical, i.e., the product $|\sigma_k(t)\pi_{\sigma k}(t)|$ has to be larger than $1/2$, the minimal value obtained for an oscillating mode [7]. For large times, $Dt \gg 1$, the product is determined by the exponentially growing Airy function $\text{Bi}(z)$,

$$|\sigma_k(t)\pi_{\sigma k}(t)| \simeq \frac{1}{\pi} D |C_B(k)|^2 \exp\left(\frac{4}{3}(Dt)^{3/2}\right). \quad (29)$$

For soft modes $k < H$, the condition $|\sigma_k(t_{dec})\pi_{\sigma k}(t_{dec})| = R_{dec} \gg 1$ yields the decoherence

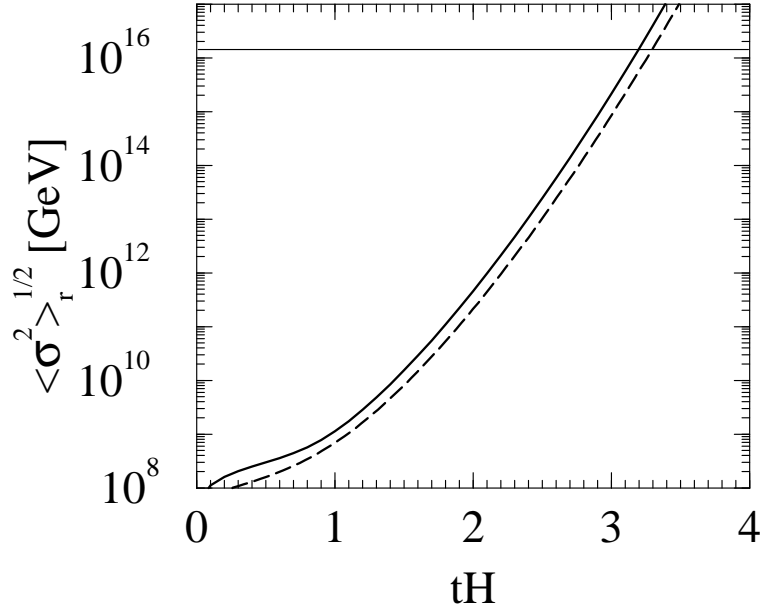


Figure 2: Growth of the variance $\langle \sigma^2(t) \rangle_r$ with time. Full and dashed lines correspond to numerical and analytical results, respectively. The horizontal line denotes the true vacuum $\langle \sigma^2 \rangle_r = 2M_G^2$.

time

$$t_{dec} \simeq \frac{1}{D} \left(\frac{3}{4} \ln(2R_{dec}) \right)^{2/3}. \quad (30)$$

For $R_{dec} = 100$ this implies $t_{dec} \sim 3/D \sim 1/H$. Results for the decoherence time obtained from the numerical integration are shown in Fig. 3.

From eq. (10) and from Fig. 3 one obtains the values of the inflaton field φ and the variance $\langle \sigma^2 \rangle_r^{1/2}$ at the decoherence time $t_{dec} \sim 3/D$,

$$M_G - \varphi_{cl}(t_{dec}) \sim 10^{12} \text{ GeV}, \quad \langle \sigma^2(t_{dec}) \rangle_r^{1/2} \sim 10^9 \text{ GeV}. \quad (31)$$

These values can serve as initial conditions for the further time evolution of the system [17]. Note, that they are different for the two fields and much larger than the frequently assumed initial condition $H/(2\pi)$.

Formation of domains

The rapid growth of tachyonic modes leads to a breakup of the homogeneous inflationary phase into domains. This spinodal decomposition is conveniently described by

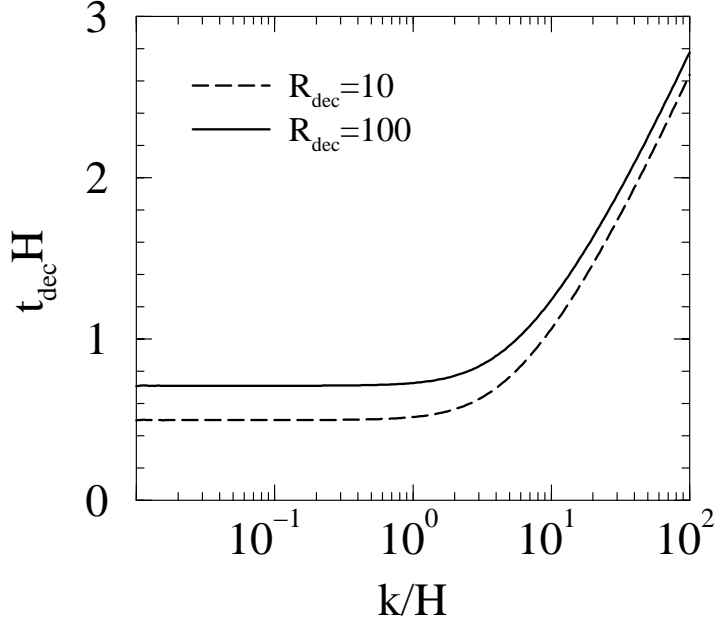


Figure 3: *Decoherence time for tachyonic modes as function of momentum.*

means of fields averaged over a volume $V_l = l^3$ characterized by some smearing function [7, 16]. For simplicity we shall use just a momentum cutoff ($k_l = 2\pi/l$),

$$\sigma_l(t, \vec{x}) = e^{-\frac{3}{2}Ht} \int_{k < k_l} \frac{d^3k}{(2\pi)^{3/2}} \left(a_\sigma(\vec{k}) \sigma_k(t) e^{i\vec{k}\vec{x}} + a_\sigma^\dagger(\vec{k}) \sigma_k^*(t) e^{-i\vec{k}\vec{x}} \right). \quad (32)$$

One then obtains for the correlation function of the smeared operator ($x = |\vec{x}|$),

$$\langle \sigma_l(t, \vec{x}) \sigma_l(t, \vec{0}) \rangle = \frac{1}{2\pi^2 x} e^{-3Ht} \int_{k < k_l} dk k |\sigma_k(t)|^2 \sin kx. \quad (33)$$

At time t the physical distance and the physical momentum are given by $y = x \exp(Ht)$ and $p = k \exp(-Ht)$, respectively.

An analytic estimate for the correlation function can be obtained based on the matching of Hankel and Airy functions described above. Consider first the case of short times, $Dt < 1$, and short distances, $1/k_l < x \ll 1/H$. Here one finds,

$$\langle \sigma_l(t, \vec{x}) \sigma_l(t, \vec{0}) \rangle \simeq -\frac{1}{4\pi^2 y^2} (\cos(k_l x) - 1). \quad (34)$$

This is the result expected for a massless scalar field in Minkowsky space. The wavelength of the oscillation is given by the ultraviolet cutoff, $l = 2\pi/k_l$.

For large times, $Dt \gg 1$, the correlation function is dominated by the tachyonic modes with $k < k_*$. For distances $x < 1/k_*$ the deviation from homogeneity is small,

$$\langle \sigma_l(t, \vec{x}) \sigma_l(t, \vec{0}) \rangle \simeq \langle \sigma^2(t) \rangle_r \left(1 + \mathcal{O}((k_* x)^2) \right). \quad (35)$$

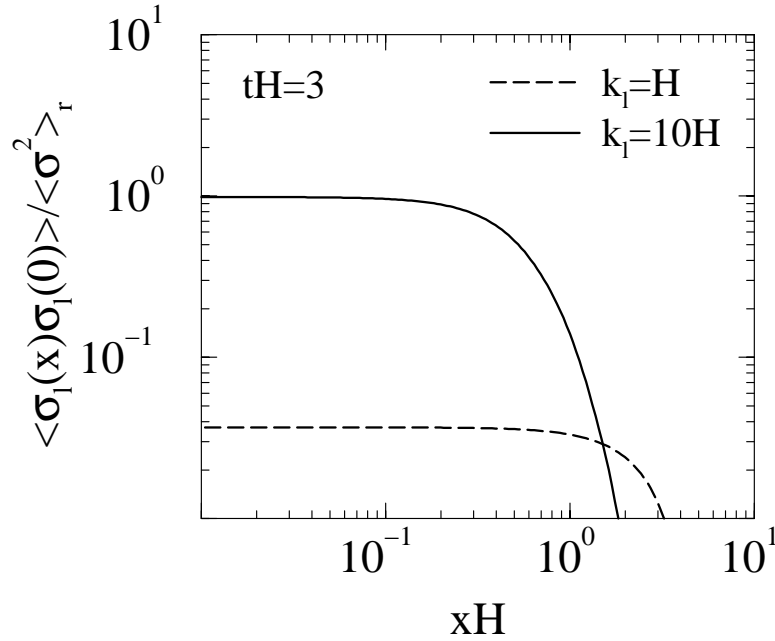


Figure 4: *Correlation function of the smeared σ -field.*

For large distances, $x > 1/k_*$, the correlation function rapidly falls off.

Numerical results for the correlation function are shown in Fig. 4 for two different values of the ultraviolet cut-off k_l . For a large cut-off, $k_l = 10H$, one reads off the correlation length $l_{corr} \simeq 1/(3H) \simeq 1/k_*$, which corresponds to the average size of a domain. For cut-offs below $1/l_{corr}$, e.g. $k_l = H$, the expected decrease of the correlation function is observed. The energy density stored in spatial variations of the σ -field is roughly given by $\rho_{grad} \sim M_G^2/l_{corr}^2$, which is much smaller than the total energy density $V \simeq \lambda^2 M_G^4$.

So far we have ignored all back reaction effects. They will terminate the slow-roll motion of the inflaton field, damp the growth of the tachyonic modes and modify the spinodal time t_{sp} . However, we do not expect a change of our qualitative picture with small decoherence time, $t_{dec} < t_{sp}$, and asymmetric classical homogeneous fields φ and σ larger than $H/(2\pi)$. The determination of the properties of the final thermal state, in particular the reheating temperature and the energy fraction of topological defects, requires further work.

Thanks to the slow-roll motion of the inflaton field φ across the critical point φ_c we have been able to follow in detail the breakup of the homogeneous inflationary phase into domains. The tachyonic modes of the field σ , which become classical before the spinodal decomposition ends, lead to an average domain size $O(1/H)$. Within a single domain they appear as a homogeneous classical field.

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